Related Rates

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How to solve related rates problems

1) **Draw a picture!**, labeling a couple of variables. **HOWEVER** do not put any numbers on your picture, except for constants! Otherwise you'll get confused later on

2) Figure out what you *ultimately* want to calculate, and don't lose track of it

3) Find an equation relating your variables

4) Differentiate your equation using the chain rule/implicit differentiation.

5) **NOW** plug in all the numbers you know! Sometimes, you might need to calculate a number of 'missing variables'. Here an extra picture as in 1), but with all the numbers plugged in, might be useful

6) Solve for whatever you were looking for in 2)

List of tricks

- Pythagorean theorem
- Definition of sin and cos
- Formulas for areas and/or volumes:
  - Volume of a cone: $V = \frac{\pi}{3} r^2 h$
  - Volume of a cylinder: $V = \pi r^2 h$
  - Volume of a ball: $V = \frac{4}{3}\pi r^3$

Problem 1

If $z = x^2 + y^2$, find $\frac{dz}{dt}$ when $x = 3$, $y = 4$, $\frac{dx}{dt} = 3$, and $\frac{dy}{dt} = -2$.

Problem 2

[3.9.19] The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is $100cm^2$?

Problem 3

[3.9.15] Two cars start at the same point. Car A travels North at a rate of 6 mi/h and Car B travels East at a rate of 2.5 mi/h. At what rate is the distance between the two cars increasing 2 hours later?

Problem 4

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall?

Problem 5

A ladder 10 feet long rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the angle between the ladder and the **wall** changing when the bottom is 6 feet from the wall?
Note: Careful! On your homework, they ask you about the angle btw the ladder and the **ground**.

Problem 6

Assume Peyam's happiness is given by $H = L^2\sqrt{M}$, where L is the number of utils (happiness points) due to teaching Math 1A lectures, and M is the number of utils due to holding office hours. If currently $L = 10$ and is increasing by 4 utils/day and $M = 100$ and is decreasing by 10 utils/day, is Peyam getting happier or sadder now, and at what rate?

Problem 7

A cylindrical gob of goo is undergoing a transformation in which its height is decreasing at a rate of 1 cm/s while its volume is decreasing at the rate of 2π cm^3/s. (It retains its cylindrical shape while all of this is happening). If, at a given instant, its volume is 24π cm^3 and its height is 6 cm, determine whether its radius is increasing or decreasing at that instant, and at what rate.